REDISTRIBUTION OF MOMENTS

in Continuous Structures

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This article focuses on the topic of moment redistribution, which should not be confused with moment distribution (that great method developed by structural engineering professor Hardy Cross for solving indeterminant structures that continues to elicit groans in structural analysis classes). Moment redistribution refers to the ability of continuous, or statically indeterminate, structures to redistribute moment at the strength limit state due to their redundancy. During the design process, analysis in the elastic range is typically performed and does not include any additional capacity of the overall structure after the elastic limit is reached. Considering moment redistribution can allow designers to account for additional capacity at the strength limit state that may be available as moment shifts (redistributes) from the section that has reached the plastic range to sections still in the elastic range.

If a continuous structure is properly detailed, it can form what is known as a "plastic hinge," which results in a shift of moment to a different area, giving a higher overall structure capacity at the strength limit state than first assumed when looking only at the section capacity. The more degrees of indeterminacy a structure has, the greater the number of potential plastic hinges required to cause failure. This concept can be illustrated by a basic example. The following assumptions will be used for the example:

- The member has an idealized moment-curvature relationship, where a section is elastic until the design capacity (yield) is reached and then does not take any additional moment.
- After a section reaches elastic capacity, it is idealized as a hinge (resisting no additional moment).
- The section capacity is the same along the entire member, with equal capacity for positive and negative applied moment.
- When a sufficient number of plastic hinges have formed to make a member or span unstable so that a collapse mechanism is formed, the member is considered to have failed.

Consider a simply supported determinant beam as shown in Fig. 1. The maximum moment under this condition is at the midspan of the beam, and, thus, that location is where the plastic moment M_p will be reached first. If the length of the beam l is 100 ft and the section moment capacity is 1000 kip-ft, the uniform load w required to form the midspan hinge and resulting collapse is calculated as follows:

$$\frac{wl^2}{8} = 1000 \text{ kip-ft}$$

$$w = \frac{8(1000 \text{ kip-ft})}{(100 \text{ ft})^2} = 0.8 \text{ kip/ft}$$

With no redundancy in this system, a collapse mechanism is formed and the member fails. This determinant beam case does not allow any redistribution of moment.

Now, consider the case of a redundant single-span member fixed on both ends with a uniform loading, as shown in Fig. 2a. For illustrative purposes, let's suppose we have a section capacity of 1000 kip-ft (both positive and negative moment) and a beam length of 100 ft. In this case, the plastic moment is reached first at the ends under negative moment. The moment diagram in Fig. 2b shows the moment values at the stage at which the first hinges form. The value of the applied uniform load w at this stage is calculated as follows:

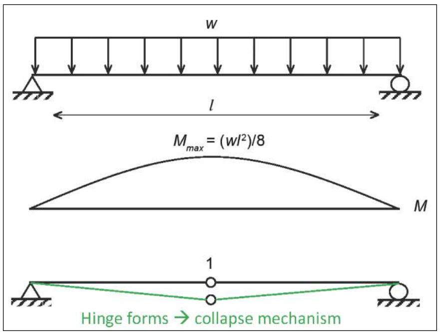

Figure 1. Simply supported determinant beam with uniform loading. All Figures: Dr. Andrea Schokker.

$$\frac{wl^2}{12} = 1000 \text{ kip-ft}$$

$$w = \frac{12(1000 \text{ kip-ft})}{(100 \text{ ft})^2} = 1.2 \text{ kip/ft}$$

The midspan moment at this stage is only 500 kip-ft. The beam is still stable and can continue to take load at this point, but no more moment can be added where the plastic hinges have formed (Fig. 2c). Any additional moment is taken as if the beam were simply supported (Fig. 2d). If we continue to add load, we will reach a point where a hinge is developed at midspan, where the maximum positive moment is. This location was already taking 500 kip-ft of moment when the hinges at the ends were formed and, once an additional moment of 500 kip-ft is added, the midspan section reaches its moment capacity of 1000 kip-ft. The incremental load Δw required to form the midspan hinge and resulting collapse is calculated as follows:

$$\frac{\Delta wl^2}{8} = 500 \text{ kip-ft}$$

$$\Delta w = \frac{8(500 \text{ kip-ft})}{(100 \text{ ft})^2} = 0.4 \text{ kip/ft}$$

This gives us a total uniform load of $w = 1.2 + 0.4 = 1.6$ kip/ft on the structure prior to reaching theoretical collapse (assuming that the hinges are formed at the elastic design capacity of the section),

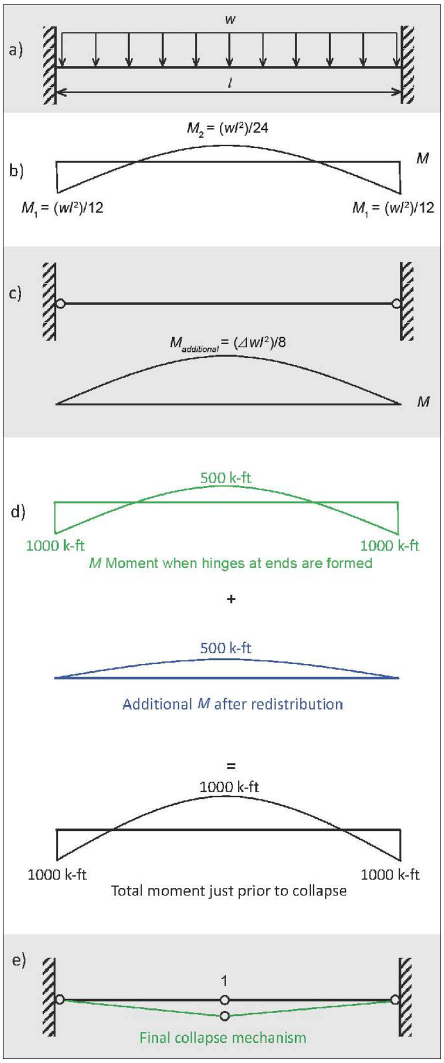

Figure 2. Redistribution of moment in a fixed-end indeterminant beam with uniform loading.

with the moment distributed as shown in Fig. 2d and the final collapse mechanism as shown in Fig. 2e. In this example, the indeterminate fixed-end beam can carry a 33% greater uniform load before theoretical collapse occurs when moment redistribution is considered.

The example gives a basic demonstration of how moments are redistributed after plastic hinges are formed. In reality, a structural concrete section will likely have different positive and negative moment capacities, based on elastic analysis.

The principles of moment redistribution are expanded in the continuous beam shown in Fig. 3a. The beam is two degrees indeterminant; therefore, it can theoretically have two hinges form prior to a third hinge that causes failure. However, the hinge location is important. If the first hinge forms in the end spans, a collapse mechanism is formed in the span (Fig. 3b). However, if the first hinge forms in an interior span, the structure can remain stable until the third hinge forms (Fig. 3c and 3d). The locations of hinges depend on a number of factors, including section capacity at different locations, span lengths, and ductility at potential hinge areas.

The American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*[1] cover moment redistribution in Article 5.6.3.4. The article states that, in lieu of a more refined analysis, redistribution (increase or decrease) of negative moment is allowed up to 20% of the moment as determined by elastic theory. The percentage is based on a factor (1000) multiplied by the net tensile strain in the steel ε_t. Redistribution is only allowed if ε_t is at least 1.5 times the tension-controlled strain limit. The article also requires that positive moments must be adjusted to account for negative moment redistribution.

Given the complexity of continuous concrete structures (particularly when including secondary moment effects, as discussed in the Summer 2018 issue of *ASPIRE*®), it is important for the designer to fully understand the implications of the redistribution of moments, as well as when they may not redistribute because of the formation of a collapse mechanism. Additionally, the section where a plastic hinge may form must be provided with sufficient ductility; therefore, particular care must be given to the detailing of reinforcement.

Ken Bondy[2] authored a helpful technical paper that goes through multiple examples with post-tensioned continuous members and builds on the basic theory of plastic hinge analysis discussed in this article. Designers are likely to find his examples instructive as they think through the moment redistribution process to create a practical concrete design.

References

1. Association of State Highway and Transportation Officials (AASHTO). 2017. *AASHTO LRFD Bridge Designs Specifications*, 8th ed. Washington, DC: AASHTO.

2. Bondy, K.B. 2003. "Moment Redistribution: Principles and Practice Using ACI 318-02." *PTI Journal*, 1(1): 3–21.

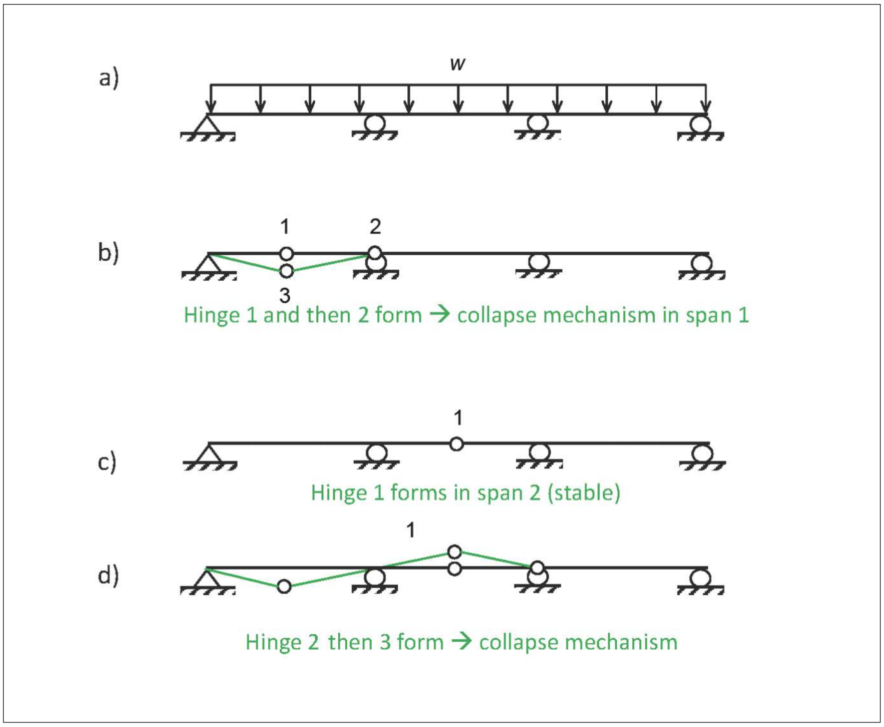

Figure 3. Possible locations of plastic hinges in a multi-span continuous beam.