How Does Cracked Concrete Carry Shear?

The modified-compression field theory (MCFT) model was introduced in the 1st edition of the AASHTO LRFD Bridge Design Specifications as a shear-resistance sectional model appropriate for both reinforced and prestressed concrete members. The MCFT of Article 5.8.3.4.2 of the LRFD Specifications is a model for predicting shear resistance of cracked concrete members.

Concrete that is subjected to shear cracks along a diagonal line. In the vast majorities of concrete, these cracks occur along an interface between the aggregate and the paste creating a rough crack surface. The crack surfaces transmit shear through the mechanics of aggregate interlock.

The variable, \( \beta \), is defined as a “factor indicating ability of diagonally cracked concrete to transmit tension and shear.” As \( \beta \) increases, the shear resistance increases. The shear carried cross the crack and acting parallel to the crack is a function of the crack width, the maximum aggregate size, and the compressive stress perpendicular to the crack.

Further, the crack width is a function of the longitudinal strain and the crack spacing. Applying equilibrium, compatibility, and constitutive relationships, the shear resistance can be determined iteratively. This is a complicated calculation not appropriate for codification. To complete the discussion, \( \Theta \), defined as the “angle of inclination of diagonal compressive stresses,” is merely a function of the longitudinal strain. As the longitudinal strain increases, \( \Theta \) increases in accordance with Equation 5.8.3.4.2-3 as shown below.

In the 1st edition of the LRFD Specifications, this complicated calculation was represented by the iterative application of simple tables of \( \beta \) and \( \Theta \). These tables now appear in Appendix B5 of the LRFD Specifications.

In the 2008 interim revisions to the LRFD Specifications, a direct calculation of \( \beta \) and \( \Theta \) was introduced, eliminating iteration. The development of these simplified provisions assumes that the shear stresses are uniformly distributed over the shear area, the direction of the principal compressive stresses remains constant with depth, and the shear resistance can be determined at a single location in the web.

The LRFD Specifications Equation 5.8.3.4.2-3 defines \( \Theta \) as:

\[
\Theta = 29 + 3500 \varepsilon_s
\]

where \( \varepsilon_s \) is the net longitudinal tensile strain at the centroid of the tension reinforcement.

For members without shear reinforcement, the LRFD Specifications Equation 5.8.3.4.2-2 defines \( \beta \) as:

\[
\beta = \frac{4.8}{(1+750\varepsilon_s)} \left( \frac{1}{29 + s_{ve}} \right)
\]

where \( s_{ve} \) is the effective crack spacing, which is a function of crack spacing and maximum aggregate size. Through the simplification, the crack width has been removed from explicit consideration, but its influence has been retained. Code writers do not typically specify or calculate crack widths so that theoretical widths do not become used as performance measures.

For members with shear reinforcement, the effective crack spacing is assumed to be 12 in. and the final term in Equation 5.8.3.4.2-2 becomes unity resulting in Equation 5.8.3.4.2-1:

\[
\beta = \frac{4.8}{(1+750\varepsilon_s)}
\]

Thus, an in-depth examination of the simplified provisions of the LRFD Specifications Article 5.8.3.4.2 reveals that the MCFT shear resistance of a cracked section is based upon its ability to transmit shear across the crack through aggregate interlock, which is a function of crack width and aggregate size. For members without transverse reinforcement such as footings and slabs, the designer must be certain that the aggregate size used in the calculation of shear resistance is the size actually used in construction because the aggregate size enters into the calculation.

Furthermore, for higher-strength concretes, where cracks tend to pass through the aggregate, aggregate size has no influence. As such, the developers of the MCFT have suggested that the maximum aggregate size be taken as zero for compressive strengths greater than 10.0 ksi. Even with this assumption, the MCFT model of the LRFD Specifications is just as accurate and conservative for members cast with specified concrete compressive strengths greater than 10.0 ksi as it is for members cast with specified strengths less than 10.0 ksi.

EDITOR’S NOTE

If you would like to have a specific provision of the AASHTO LRFD Bridge Design Specifications explained in this series of articles, please contact us at www.aspirebridge.org.